U.Porto Journal of Engineering, 8:2 (2022) 82-94
ISSN 2183-6493
DOI: 10.24840/2183-6493_008.002_0007

Received: 31 October, 2021
Accepted: 3 January, 2022
Published: 28 April, 2022

Some Studies on Verifying the Applicability of Free Vibration-based Modes Shapes Method to Rectangular Shaped Cracks in a Cantilever Beam

V. Khalkar[1], S. G. Kumbhar[2], P. Hariharasakhtisudhan[3], A. A. M. Moshi[4], S. D. Jadhav[5], N. Kumbhar[6], P. Oak[7], P. S. Joshi[8]

[1]Department of Mechanical Engineering, Gharda Institute of Technology, Lavel, Khed, Ratnagiri, India (vikas_khalkar@rediffmail.com); [2]Department of Automobile Engineering, Rajarambapu Institute of Technology, Islampur, Sangli, India (surajkumar.kumbhar@ritindia.edu); [3]Department of Mechanical Engineering, National Engineering College, Kovilpatti, India (harimeed2012@gmail.com); [4]Department of Mechanical Engineering, National Engineering College, Kovilpatti, India (moshibeo2010@gmail.com); [5]Department of Mechanical Engineering, Bharati Vidyapeeth College of Engineering, Navi Mumbai, Mumbai, India (sandhya.jadhav@bhartividyapeeth.edu); [6]Department of Mechanical Engineering, Gharda Institute of Technology, Lavel, Khed, Ratnagiri, India (bkumbhar2050@gmail.com); [7]Department of Computer Engineering, Gharda Institute of Technology, Lavel, Khed, Ratnagiri, India (pvoak@git-india.edu.in); [8]Department of Mechanical Engineering, Gharda Institute of Technology, Lavel, Khed, Ratnagiri, India (pramodsjoshi29@gmail.com)

Abstract

Cracks weaken structures. When the crack size increases in service, the structure becomes weaker than its earlier condition. Lastly, the structure may break down due to a small crack. Therefore, crack detection and classification is a fundamental issue. Many aspects of defects have already been addressed, but the application of non-destructive testing methods to structural materials has become more widespread. For a long time, vibration methods based on Natural Frequency and Mode Shapes have been used for possible cracks detection in the beams. The impact of arbitrary and random defect geometry on applying these methods has been overlooked. This study focuses on a mode shapes-based vibration analysis of a cracked cantilever beam to investigate this issue. The effects of crack geometries on mode shapes are examined theoretically and numerically using a new crack model (Rectangular shaped crack), which differs from the well-known V-shaped crack. A MATLAB code is written to obtain the natural frequencies and mode shapes for all cracked instances of beams. The mode shapes result of both the new (Rectangular), and V-shaped models are compared, and it is found that the results are less sensitive to the geometry change.

Author Keywords. Natural Frequency, V-shaped Crack, Mode Shapes, MATLAB, ANSYS, Rectangular Shaped Crack.

Type: Research Article

1. Introduction

Steel materials are employed in several structural applications in the civil, automotive, and aerospace industries. Steel beams are a typical use of steel in construction. The structures are designed to work safely for the remainder of their useful life. On the other hand, damage begins at the point where the structure breaks down. The crack affects the modal and physical properties of the beam, such as mode shapes, natural frequency, damping, and stiffness. As a result, the beam's dynamic reaction changes dramatically. It is necessary to regularly monitor the condition of beams or structures to avoid catastrophic collapse in applications. Free vibration-based approaches are used for identifying cracks in beams for a long time. For probable fracture identification in beams, these approaches leverage the effect of mode shapes, damping, stiffness, and natural frequency as an input in the inverse problem of vibration analysis. The dynamic behavior of a cracked cantilever beam for breathing notch was examined by Gawande and More (2016). They had developed theoretical equations for evaluating mode shapes and natural frequencies using a systematic approach. The relationship between the natural modal frequency and fracture characteristics were described using the method offered in this study report. The influence of fracture crack and depth location on the cantilever beam’s natural frequency was studied (Barad, Sharma, and Vyas 2013). Natural frequency was employed as a starting point for identifying crack parameters in this research. Aye and Htike (2019) employed the first two relative natural frequencies to forecast the cantilever beam's crack characteristics. They examined the broken cantilever beams for the first two natural frequencies using experiments. They created a correlation model to determine the locations and depths of cracks in cantilever beams. The influence of crack geometry on the natural frequency of composite beams was investigated by Orhan et al. (2016). They were thinking of a V-shaped crack with a novel crack model. According to the findings of their study, natural frequencies are not affected by changes in geometry. Ostachowicz and Krewczuk (1991) proposed a method for determining the crack's location based on the beam's deflection form. Rizos, Aspragathos, and Dimarogonas (1990) assessed the amplitude at two sites and suggested a method to find the crack's location. Pandey and Biswas (1994) proposed the curvature of deflected form as a measure for identifying the location of cracks in beams. Lakshmi Narayana and Jebaraj (1999) investigated the effect of cracks on the behavior of mode shape using analytical methods. Kishen and Sain (2004) developed a static data-based method for detecting damage. The experimental and analytical approach for crack identification in cantilever beams was established by Nahvi and Jabbari (2005). Marques, Vandepitte, and Tita (2021) presented the methodology for structural health monitoring fatigue fractures under static random loads. The technique is divided into two sections: an empirical correlation for tracking the evolution of damage indices and a numerical system for estimating fatigue life. The damage indices are calculated using the variation in the structure's vibration response due to fracture propagation. Two damage metrics are investigated for a cantilever beam subjected to random base excitation, and the approach is verified. The case study supports the methodology's usefulness and demonstrates that both measures have the potential to detect damage. The suggested framework uses a frequency domain method to determine the probability density function of the stress and an equivalent stress approach based on Walker's equation for fatigue fracture growth to calculate the fatigue life numerically. Khalkar and Logesh (2020) studied the vibration-based mode shapes method's applicability for the possible crack detection in the cantilever beams. They considered different kinds of practical cracks on the beam, i.e., V-shaped and U-shaped. The dynamic behavior of a broken beam was examined by Khalkar and Ramachandran (2018, 2019). They have discussed various topics, including the impact of fracture shape and position on dynamic behavior. They also looked at applying current mathematical models to different fracture geometries. Their study adds to the existing damage measurement literature in a significant way. Stress-corroded turbine blades result in a material loss that is localized. The damage site can be mapped into the Rectangular shaped open-edged fractures (Khalkar and Ramachandran 2018) once the localized materials have been removed from the blade. As a result, a novel fracture mode, i.e., rectangular-shaped crack model.

Because of this, the Rectangular shaped crack model was used in this study, along with a traditional V-shaped crack model, to evaluate the possibility of crack detection in cantilever beams using a free vibration mode shapes-based technique. These tests were performed using numerical and theoretical analysis while subjected to free vibration loading.

2. Simulated Crack Configurations

Two case studies were considered in this research study, i.e., Case study A and Case study B.

Case study A: To analyze the influence of cracks on natural frequency and the mode shapes, a total of eighteen cracked specimens were used. Nine cracked specimens had V-shaped open-edged cracks, whereas the remaining nine had Rectangular shaped open-edged cracks. Case A-1 and Case A-2 were considered as two different cracked cases.

Case A-1: This case had a total of nine V-shaped crack specimens. In this case, the investigation was further subdivided into three more sub-cases. The first sub-case, where an 80 mm crack was selected from the cantilevered end and crack depth was adjusted from 5 mm to 15 mm by 5 mm interval. The second and third sub-instances were nearly identical. The only variation was that 160 mm and 240 mm crack locations were chosen for the second and third sub instances instead of 80 mm crack locations. Figure 1 depicts a single V-shaped broken instance of a cantilever beam.

Case A-2: This case was similar to case A-1, except that rectangular-shaped cracks were investigated on the cantilever beams instead of V-shaped cracks. Figure 2 depicts a single rectangular-shaped cracked case of a cantilever beam.

Case study B: In this research study, to verify the results of theoretical models, the various cracked cases of the previous study (Khalkar and Logesh 2020) were considered. In this case study, nine specimens of EN 8 materials of V-shaped cracks were considered. Again, this case study was divided into three sub-cases, i.e., sub-case B-1, sub-case B-2, and sub-case B-3. In the first sub-case, 80 mm crack location was chosen from the cantilevered end, and at this location, the crack depth was varied from 5 mm to 15 mm by an interval of 5 mm. The second and third sub-cases were similar to that of the first sub-case; the only difference was that instead of 80 mm crack location, 160 mm and 240 mm crack locations were selected from the cantilevered end of the beam.

3. Theoretical Methods

When a cantilevered beam vibrates freely at a given time, just the beam's effective mass (meff) is in play, which causes the vibrations. The total mass and effective mass (Khalkar and Ramachandran 2018, 2019) of the beam were calculated from prior research and are shown in Table 1. In this research investigation, the fractured cantilever beam was converted into discrete structures (Radhakrishnan 2004), which included two springs and two mass systems. The magnitudes of two lumped masses, as given in Table 1. For the portion of the cantilever beam between the cantilevered end and the crack (Radhakrishnan 2004), K1 stiffness was used. For the segment (Radhakrishnan 2004) between the free end and the crack of the same beam, K2 stiffness was selected. The stiffness of the beam is also depicted in Figure 3. K1 and K2 are the stiffness's of the un-cracked and cracked beams, respectively.

Total Beam mass	Beam effective mass	m1= m2= meff/2
1.12608 (kg)	0.2654 (kg)	0.1327(kg)

Table 1: The beam's material properties

Then for Figure 3, Newton's law of motion was applied and received two equations: (1) and (2).

$$m_1 X_1'' = -K_1 X_1 - K_2\,(X_1 - X_2)$$
$$m_1 X_1'' + K_1 X_1 + K_2 (X_1 - X_2) = 0 \quad (1)$$
$$m_1 X_1'' + (K_1 + K_2)\,X_1 - K_2 X_2 = 0$$

$$m_2 X_2'' = -K_2\,(X_2 - X_1) \quad (2)$$
$$m_2 X_2'' + K_2 X_2 - K_2 X_1 = 0$$

Equation (1) and Equation (2) were used to create a global mass matrix, which is represented in Equation (3)

$$m = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \quad (3)$$

A global square stiffness matrix is obtained using Equation (1) and Equation (2), and it is shown by Equation (4)

$$K = \begin{bmatrix} K_1 + K_2 & -K_2 \\ -K_2 & K_2 \end{bmatrix} \quad (4)$$

The system matrix was created using Equation (3) and Equation (4) in the MATLAB application.

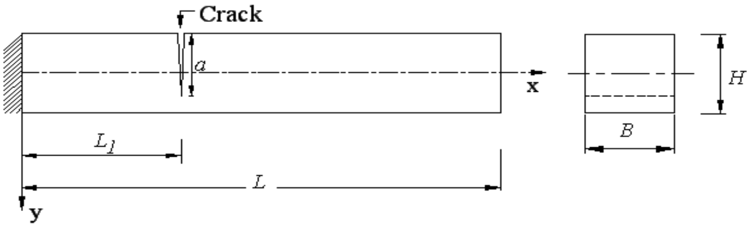

Figure 1: V-shaped open edged cracked cantilever beam

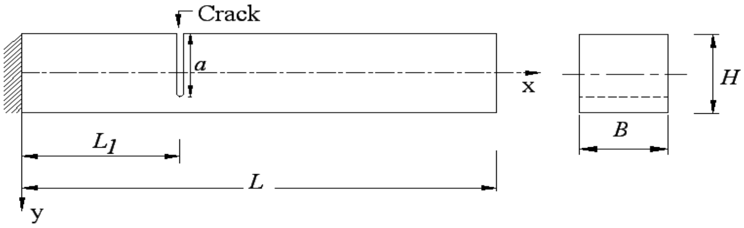

Figure 2: A Rectangular-shaped open edged cracked cantilever beam

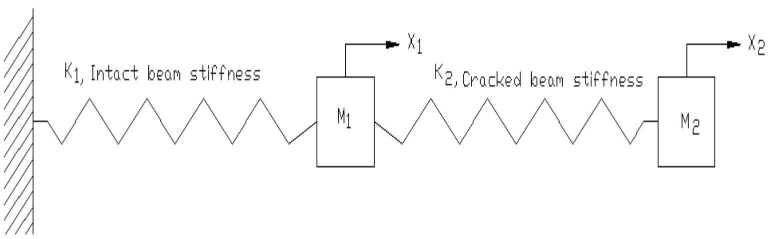

Figure 3: Discrete model of a crack cantilever beam

4. Numerical Modeling

The mode shapes of cracked cantilever beams were determined with the help of ANSYS 12.1 finite element software. First, a cracked cantilever beam's effective mass was estimated (Khalkar and Ramachandran 2019). Because the difference in mass between un-cracked and cracked beams with the highest crack depth was negligibly slight in this investigation, the mass of intact and cracked beams was assumed to be the same. The model was then changed from a continuous cracked cantilever beam to a discrete (Radhakrishnan 2004) model. The discrete model is shown in Figure 4. The discrete model carried two lines and two key points. Figure 4 depicts this. Mass 21 and combination 14 elements were used in the cracked beams vibration analysis. The lumped masses and springs were represented as key points and lines. Figures 4 to 7 show the natural frequencies and mode shapes graphs for various fractured instances.

Figure 4: First mode shape plot; V-shaped crack; crack depth 5 mm; location 80 mm

Figure 5: Second mode shape plot; V-shaped crack; crack depth 5 mm; location 80 mm

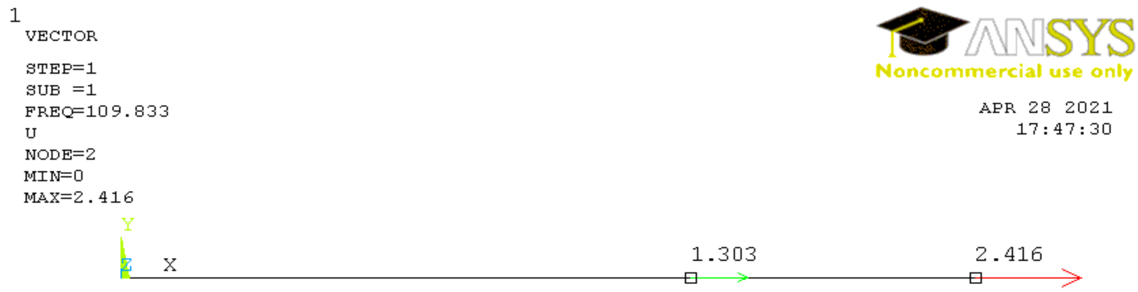

Figure 6: First mode shape plot; Rectangular-shaped crack; crack depth 15 mm; location 240 mm

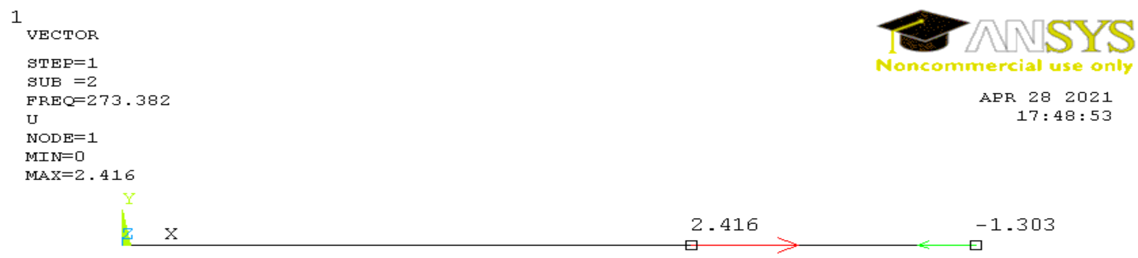

Figure 7: Second mode shape plot; Rectangular-shaped crack; crack depth 15 mm; location 240 mm

5. MATLAB Programming

A code is written in MATLAB software to determine cracked beams' natural frequency and mode shapes. The global mass matrix and global stiffness matrix shown in Equation (3) and Equation (4) have been considered in this program. A MATLAB code is given here.

```
% Programming in MATLAB to obtain
% themode shapes and natural frequencies of a cracked cantilever beam
% Define [K] and [M] matrices
M=[0.1327 0;0 0.1327]
K=[317558.28 -137174.2; -137174.2 137174.2]
% Form the system matrix
A=inv (M)*K
% Obtain eigenvalues and eigenvectors of A
[V, D]=eig (A)
 %V and D above are matrices.
%V-matrix gives the eigenvectors and
 %the diagonal of D-matrix gives the eigen-values
% Sort eigen-values and eigen-vectors
[D_sorted, ind] = sort (diag(D),'ascend');
V_sorted = V (:,ind);
%Obtain natural frequencies and mode shapes
 nat_freq_1 = sqrt (D_sorted(1))
 nat_freq_2 = sqrt (D_sorted(2))
mode_shape_1 = V_sorted(:,1)
mode_shape_2 = V_sorted(:,2)
```

6. Results and Discussion

In this research study, to ensure the validity of the developed theoretical models, the nine cracked cases of the previous study (Khalkar and Logesh 2020) were considered and solved to get the theoretical mode shapes. Table 2 shows the mode shapes of various cracked cases of a cantilever beam. The theoretical and experimental results for the mode shapes gave good agreement. The stiffness of cracked cantilever beams shown in Table 3 has been taken (Khalkar and Ramachandran 2018) from the previous study. In this case study, the natural frequencies and mode shapes of various cracked cases are investigated using the theoretical and numerical (ANSYS 2012) methods.

The theoretical natural frequencies and mode shapes for V-shaped cracked cases and Rectangular ones are presented in Table 4 and Table 5, respectively. Similarly, the numerical (FEA) natural frequencies and mode shapes for V-shaped cracked cases and Rectangular-shaped cracked cases are presented in Table 6 and Table 7, respectively. The numerical and theoretical results strongly agree with the natural frequencies and the mode shapes. Table 8 and Table 9 show the percent variation in mode shapes frequency between V-shaped and rectangular cracked cases for spring cracked cantilever beams.

Crack location ratio (L1/L)	**Crack depth ratio (a/H)**	**Methods**	**Mode shapes**		
			X1	X2	X2/X1
0.222	0.25	Theoretical method	-0.5169	-0.8561	1.656
0.222	0.25	Experimental method (Khalkar and Logesh 2020)	2.605	4.208	1.615
		Percentage error			2.475
0.222	0.5	Theoretical method	-0.4779	-0.8784	1.838
0.222	0.5	Experimental method (Khalkar and Logesh 2020)	258.9	432.36	1.669
		Percentage error			9.194
0.222	0.75	Theoretical method	-0.3290	-0.9443	2.87
0.222	0.75	Experimental method (Khalkar and Logesh 2020)	257.98	754.37	2.922
		Percentage error			-1.811
0.444	0.25	Theoretical method	-0.5211	-0.8535	1.637
0.444	0.25	Experimental method (Khalkar and Logesh 2020)	740.14	1215	1.641
		Percentage error			-0.244
0.444	0.5	Theoretical method	-0.5005	-0.8657	1.729
0.444	0.5	Experimental method (Khalkar and Logesh 2020)	243.97	458.14	1.877
		Percentage error			-8.559
0.444	0.75	Theoretical method	-0.4133	-0.9106	2.203
0.444	0.75	Experimental method (Khalkar and Logesh 2020)	28.52	66.93	2.346
		Percentage error			-6.491
0.666	0.25	Theoretical method	-0.5243	-0.8515	1.624
0.666	0.25	Experimental method (Khalkar and Logesh 2020)	398.03	708.56	1.78
		Percentage error			-9.605
0.666	0.5	Theoretical method	-0.5167	-0.8562	1.657
0.666	0.5	Experimental method (Khalkar and Logesh 2020)	294.57	515.52	1.75
		Percentage error			-5.612
0.666	0.75	Theoretical method	-0.4805	-0.877	1.825
0.666	0.75	Experimental method (Khalkar and Logesh 2020)	437.41	746.9	1.707
		Percentage error			6.465

Table 2: Mode shapes of a cracked cantilever beam (V-shaped cracked cases)

Crack location (mm)	Crack depth (mm)	Stiffness (N/m)	
		V-shaped crack	Rectangular shaped crack
80	5	171526.1	170648.5
80	10	139470	136925.7
80	15	71581.96	67024.13
160	5	175746.9	175131.4
160	10	156739.8	155279.5
160	15	103092.8	96899.22
240	5	178890.9	178571.4
240	10	171232.9	170357.8
240	15	141242.9	137174.2

Table 3: Stiffness of a cracked cantilever beam (Khalkar and Ramachandran 2018)

80 mm crack location of a V-shaped crack from the cantilevered end				
	Natural frequencies (rad/sec or Hz)		**Mode shapes**	
Crack depth (mm)	ωn1 or fn1	ωn2 or fn2	(X2/X1) first	(X2/X1) second
5	715.4498 or 113..86	1852.7 or 294.86	(-0.8560)/(-0.5170)= 1.655	0.5170/(-0.8560)= -0.603
10	692.13 or 110.15	1726.9 or 274.84	(-0.8784)/(-0.4780) =1.837	0.4780/(-0.8784)= -0.544
15	592.77 or 94.34	1444.6 or 229.91	(-0.9443)/(0.3292)= 2.868	0.3292/(-0.9443)= -0.348
160 mm crack location of a V-shaped crack from the cantilevered end				
5	717.94 or 114.26	1868.9 or 297.44	(-0.8534/-0.5213) =1.637	(0.5213/-0.8534)= -0.61
10	705.74 or 112.32	1795.4 or 285.74	(-0.8657/-0.5006) =1.729	(0.5006/-0.8657)= -0.578
15	651.40 or 103.67	1577.6 or 251.08	(-0.9106/-0.4133) =2.203	(0.4133/-0.9106)= -0.453
240 mm crack location of a V-shaped crack from the cantilevered end				
5	719.73 or 114.54	1880.8 or 299.33	(-0.8515/-0.5243) = 1.624	(0.5243/-0.8515)= -0.615
10	715.27 or 113.83	1851.6 or 294.69	(-0.8562/-0.5167)=1.657	0.5167/(-0.8562)= -0.603
15	693.66 or 110.39	1734 or 275.97	(-0.877/-0.4805)=1.825	(0.4805/-0.877)= -0.547

Table 4: Theoretical method-based mode shapes and natural frequencies of a cracked cantilever beam (V-shaped crack)

80 mm crack location of a Rectangular-shaped crack from the cantilevered end				
	Natural frequencies (rad/sec or Hz)		**Mode shapes**	
Crack depth (mm)	ωn1 or fn1	ωn2 or fn2	(X2/X1) first	(X2/X1) second
5	714.91 or 113.78	1849.4 or 294.34	(-0.8565/-0.5161) =1.659	0.5161/(-0.8565)= -0.602
10	689.88 or 109.8	1716.7 or 273.22	(-0.8804/-0.4743) = 1.856	0.4743/(-0.8804)= -0.538
15	581.34 or 92.52	1425.3 or 226.85	(-0.9494/-0.3141) =3.022	0.3141/(-0.9494)= -0.33
160 mm crack location of a Rectangular-shaped crack from the cantilevered end				
5	717.59 or 114.21	1866.5 or 297.07	(-0.8538/-0.5206)= 1.64	0.5206/(-0.8538)= -0.609
10	704.699 or 112.15	1789.7 or 284.84	(-0.8667/-0.4989)= 1.737	0.4989/(-0.8667)= -0.575
15	642.109 or 102.19	1551.6 or 246.95	(-0.9169/-0.3992)=2.296	0.3992/(-0.9169))= -0.435
240 mm crack location of a Rectangular-shaped crack from the cantilevered end				
5	719.55 or 114.52	1879.6 or 299.15	(-0.8517/-0.5240)=1.625	0.5240/(-0.8517)= -0. 615
10	714.73 or 113.75	1848.3 or 294.17	(-0.8567/-0.5158)= 1.66	0.5158/(-0.8567)= -0.602
15	690.1 or 109.83	1717.7 or 273.38	(-0.8802/-0.4747)= 1.854	0.4747/(-0.8802)= -0.539

Table 5: Theoretical method-based mode shapes and natural frequencies of a cracked cantilever beam (Rectangular shaped crack)

80 mm crack location of a V-shaped crack from the cantilevered end				
	Natural frequencies (Hz)		**Mode shapes**	
Crack depth (mm)	fn1	fn2	(X2/X1) first	(X2/X1) second
5	113.87	294.87	2.3498/1.4193= 1.655	(-1.4193)/2.3498 = -0.604
10	110.16	274.85	2.4112/1.3122 = 1.837	(-1.3122)/2.4112 = -0.544
15	94.343	229.91	2.5921/0.90364 = 2.868	(-0.90364)/2.5921 = -0.348
160 mm crack location of a V-shaped crack from the cantilevered end				
5	114.26	297.44	2.3427/1.4309 = 1.637	(-1.4309)/2.3427 = - 0.61
10	112.32	285.75	2.3764/1.3743 = 1.729	(-1.3743)/2.3764 = -0.5783
15	103.67	251.08	2.4998/1.1344 = 2.203	(-1.1344)/2.4998 = -0.453
240 mm crack location of a V-shaped crack from the cantilevered end				
5	114.55	299.34	2.3376/1.4393 = 1.624	(-1.4393)/2.3376 = - 0.615
10	113.84	294.69	2.3503/1.4184 = 1.657	(-1.4184)/2.3503 = - 0.603
15	110.40	275.98	2.4074/1.3191 = 1.825	(-1.3191)/2.4074 = - 0.547

Table 6: Mode shapes and natural frequencies of a cracked cantilever beam (V-shaped crack) using FEA package

80 mm crack location of a Rectangular-shaped crack from the cantilevered end				
	Natural frequencies (Hz)		**Mode shapes**	
Crack depth (mm)	fn1	fn2	(X2/X1) first	(X2/X1) second
5	113.78	294.34	2.3513/1.4168 = 1.659	(-1.4168)/2.3513 = -0.602
10	109.8	273.22	2.4167/1.3020 = 1.856	(-1.3020)/2.4167 = - 0.538
15	92.523	226.85	2.6062/0.86234 = 3.022	(-0.86234)/2.6062 = -0.33
160 mm crack location of a Rectangular-shaped crack from the cantilevered end				
5	114.21	297.07	2.3437/1.4293 = 1.639	(-1.4293)/2.3437 = -0.609
10	112.16	284.84	2.3792/1.3695 = 1.737	(-1.3695)/2.3792 = -0.575
15	102.19	246.94	2.517/1.0958 = 2.297	(-1.0958)/2.517 = -0. 435
240 mm crack location of a Rectangular-shaped crack from the cantilevered end				
5	114.52	299.15	2.3381/1.4385 = 1.625	(-1.4385)/2.3381 = - 0.615
10	113.75	294.16	2.3518/1.4159 = 1.66	(-1.4159)/2.3518 = - 0.602
15	109.83	273.38	2.4132/1.3030 = 1.852	(-1.3030)/2.4132 = - 0.539

Table 7: Mode shapes and natural frequencies of a cracked cantilever beam (Rectangular –shaped crack) using FEA package

80 mm crack location from the cantilevered end						
	First mode shape (X2/X1)			**Second mode shape (X2/X1)**		
Crack depth (mm)	V-shaped crack	Rectangular shaped crack	% variation	V-shaped crack	Rectangular shaped crack	% variation
5	1.655	1.659	0.241	-0.603	-0.602	-0.166
10	1.837	1.856	1.023	-0.544	-0.538	-1.115
15	2.868	3.022	5.09	-0.348	-0.33	-5.454
160 mm crack location from the cantilevered end						
5	1.637	1.64	0.182	-0.61	-0.609	-0.164
10	1.729	1.737	0.46	-0.578	-0.575	-0.521
15	2.203	2.296	4.05	-0.453	-0.435	-4.137
240 mm crack location from the cantilevered end						
5	1.624	1.625	0.0615	-0.615	-0.615	0
10	1.657	1.66	0.1807	-0.603	-0.602	-0.166
15	1.825	1.854	1.564	- 0.547	-0.539	-1.484

Table 8: Mode shapes of a cracked cantilever beam computed by theoretical method

80 mm crack location from the cantilevered end						
	First mode shape (X2/X1)			Second mode shape (X2/X1)		
Crack depth (mm)	V-shaped crack	Rectangular shaped crack	% variation	V-shaped crack	Rectangular shaped crack	% variation
5	1.655	1.659	0.241	-0.604	-0.602	-0.332
10	1.837	1.856	1.023	-0.544	-0.538	-1.11
15	2.868	3.022	5.095	-0.348	-0.33	-5.45
160 mm crack location from the cantilevered end						
5	1.637	1.639	0.122	-0.61	-0.609	-0.164
10	1.729	1.737	0.46	-0.5783	-0.575	-0.573
15	2.203	2.297	4.092	-0.453	-0.435	-4.137
240 mm crack location from the cantilevered end						
5	1.624	1.625	0.061	-0.615	-0.615	0
10	1.657	1.66	0.1807	-0.603	-0.602	-0.166
15	1.825	1.852	1.457	-0.547	-0.539	-1.484

Table 9: Mode shapes of a cracked cantilever beam computed by FEA method

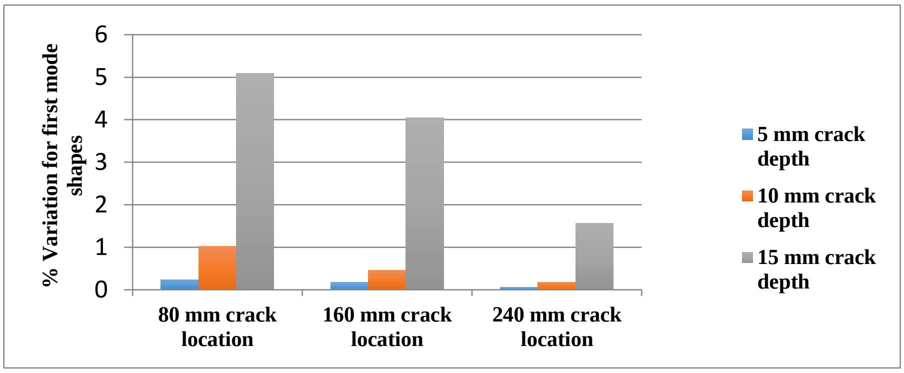

Figure 8: % variation for first mode shapes between V-shaped and Rectangular-shaped cracked cases by theoretical method

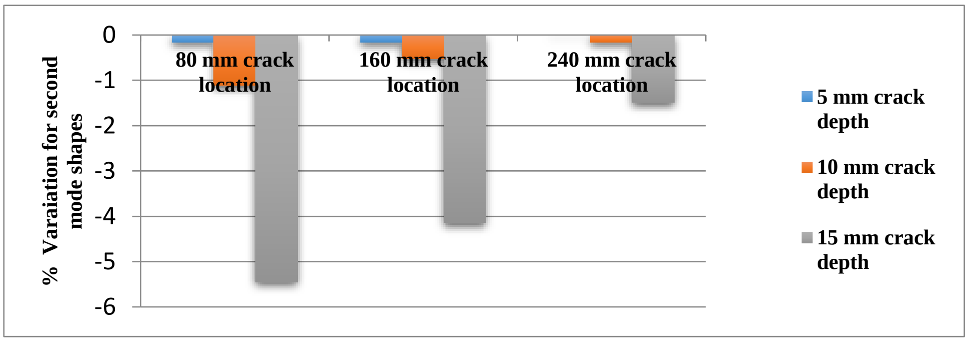

Figure 9: % variation for second mode shapes between V-shaped and Rectangular-shaped cracked cases by theoretical method

Figure 8 and Figure 9 show the percentage difference in mode shapes between V-shaped and Rectangular cracked cases for various crack locations. Figure 8 and Figure 9 show that the percentage variation in mode shapes between V-shaped and Rectangular-shaped cracked cases is less than 5.454 percent for all cracked specimens. The difference in mode shapes induced by crack geometry changes is only a minimal effect. As a result, steel materials are slightly susceptible to crack geometries changes in vibration properties such as mode shapes. When crack identification is based on free vibration, the influence of mode shapes has long been used as a primary criterion. Regardless of crack geometries, such as V-shaped cracks and rectangular cracks, the mode shapes-based free vibration method can reliably predict the location and depth of cracks in structures.

7. Conclusions

Numerical and theoretical methods were used to analyze the steel cracked beam's free vibrations with various crack locations, depths, and geometries. Several findings can be made from a free vibration investigation of a cracked cantilever beam:

Regardless of the crack geometries, the free vibration multiple mode shapes-based crack detection method can reliably predict crack locations and depths in structures.

By treating the beam as a two mass-two spring system or discrete systems, the first two natural frequencies and mode forms of a broken cantilever beam can be efficiently investigated, since the crack depth ratio (a/H) is higher than 0.5.

When the fracture geometry changes from V-shaped to rectangular, there is a slight alteration in the cracked beam's mode shapes.

In combination with FEA, vibration monitoring on such structures can help examine the degree of flaws in the structures.

References

ANSYS. 2012. *ANSYS Release 12.1*. ANSYS Inc.

Aye, W. P. P., and T. M. Htike. 2019. "Inverse method for identification of edge crack using correlation model". *SN Applied Sciences* 1, no. 6: Article number 590. https://doi.org/10.1007/s42452-019-0618-x.

Barad, K. H., D. S. Sharma, and V. Vyas. 2013. "Crack detection in cantilever beam by frequency based method". *Procedia Engineering* 51: 770-75. https://doi.org/10.1016/j.proeng.2013.01.110.

Gawande, S. H., and R. R. More. 2016. "Effect of notch depth & location on modal natural frequency of cantilever beams". *Structures* 8: 121-29. https://doi.org/10.1016/j.istruc.2016.09.003.

Khalkar, V., and S. Ramachandran. 2018. "The effect of crack geometry on stiffness of spring steel cantilever beam". *Journal of Low Frequency Noise Vibration and Active Control* 37, no. 4: 762-73. https://doi.org/10.1177/1461348418765959.

———. 2019. "The effect of crack geometry on non-destructive fault detection of EN 8 and EN 47 cracked cantilever beam". *Noise and Vibration Worldwide* 50, no. 3: 92-100. https://doi.org/10.1177/0957456519834537.

Khalkar, V., and K. Logesh. 2020. "The effect of crack geometry on mode shapes of a cracked cantilever beam". *Australian Journal of Mechanical Engineering*: 1-12. https://doi.org/10.1080/14484846.2020.1766349.

Kishen, J. M. C., and T. Sain. 2004. "Damage detection using static test data". *Journal of Structural Engineering* 31, no. 1: 15-21.

Lakshmi Narayana, K., and C. Jebaraj. 1999. "Sensitivity analysis of local/global modal parameters for identification of a crack in a beam". *Journal of Sound and Vibration* 228, no. 5: 977-94. https://doi.org/10.1006/jsvi.1999.2445.

Marques, D. E. T., D. Vandepitte, and V. Tita. 2021. "Damage detection and fatigue life estimation under random loads: A new structural health monitoring methodology in the frequency domain". *Fatigue & Fracture of Engineering Materials & Structures* 44, no. 6: 1622-36. https://doi.org/10.1111/ffe.13439.

Nahvi, H., and M. Jabbari. 2005. "Crack detection in beams using experimental modal data and finite element model". *International Journal of Mechanical Sciences* 47, no. 10: 1477-97. https://doi.org/10.1016/j.ijmecsci.2005.06.008.

Orhan, S., M. Lüy, M. H. Dirikolu, and G. M. Zorlu. 2016. "The effect of crack geometry on the nondestructive fault detection in a composite beam". *International Journal of Acoustics and Vibrations* 21, no. 3: 271-73. https://doi.org/10.20855/ijav.2016.21.3420.

Ostachowicz, W. M., and M. Krawczuk. 1991. "Analysis of the effect of cracks on the natural frequencies of a cantilever beam". *Journal of Sound and Vibration* 150, no. 2: 191-201. https://doi.org/10.1016/0022-460X(91)90615-Q.

Pandey, A. K., and M. Biswas. 1994. "Damage detection in structures using changes in flexibility". *Journal of Sound and Vibration* 169, no. 1: 3-17. https://doi.org/10.1006/jsvi.1994.1002.

Radhakrishnan, V. M. 2004. "Response of a cracked cantilever beam to free and forced vibrations". *Defence Science Journal* 54, no. 1: 31-38. https://doi.org/10.14429/dsj.54.2019.

Rizos, P. F., N. Aspragathos, and A. D. Dimarogonas. 1990. "Identification of crack location and magnitude in a cantilever beam from the vibration modes". *Journal of Sound and Vibration* 138, no. 3: 381-88. https://doi.org/10.1016/0022-460X(90)90593-O.